

# VECM Analysis in JMulTi

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Vector error correction models (VECMs) with a prespecified number of cointegrating relations, a finite number of lagged differences, deterministic terms and exogenous variables can be specified, estimated and used for forecasting, causality and impulse response analysis in JMulTi . The relevant features are described in the following. For further information and references see Chapter 3 of Lütkepohl and Krätzig (2004).

# 1 The Model

The general setup of a VECM allowed for in JMULTi is of the form

$$\Gamma_0 \Delta y_t = \alpha [\beta' : \eta'] \begin{bmatrix} y_{t-1} \\ D_{t-1}^{co} \end{bmatrix} + \Gamma_1 \Delta y_{t-1} + \dots + \Gamma_p \Delta y_{t-p} + B_0 x_t + \dots + B_q x_{t-q} + C D_t + u_t, \quad (1)$$

where  $y_t = (y_{1t}, \dots, y_{Kt})'$  is a vector of  $K$  observable endogenous variables,  $x_t = (x_{1t}, \dots, x_{Mt})'$  is a vector of  $M$  observable exogenous or unmodelled variables,  $D_t^{co}$  contains all deterministic terms included in the cointegration relations and  $D_t$  contains all remaining deterministic variables. Deterministic variables may be constants, linear trends, seasonal dummy variables as well as user specified other dummy variables. Notice that a single deterministic term cannot appear in both  $D_t$  and  $D_t^{co}$  so that the two vectors have to contain mutually exclusive terms. The residual vector  $u_t$  is assumed to be a  $K$ -dimensional unobservable zero mean white noise process with positive definite covariance matrix  $E(u_t u_t') = \Sigma_u$ .

The parameter matrices  $\alpha$  and  $\beta$  have dimensions  $(K \times r)$  and they have to have rank  $r$ . They specify the long-run part of the model with  $\beta$  containing the *cointegrating relations* and  $\alpha$  representing the *loading coefficients*. The column dimension of  $\eta$  is also  $r$  and its row dimension corresponds to the dimension of  $D_t^{co}$ . The notation

$$\beta^* = \begin{bmatrix} \beta \\ \eta \end{bmatrix}$$

will be used in the following and the row dimension of  $\beta^*$  will be denoted by  $K^*$ . Hence,  $\beta^*$  is a  $(K^* \times r)$  matrix.

The cointegrating rank  $r$  has to be specified by the user. It has to be in the range  $1 \leq r \leq K - 1$ . Cointegration tests for determining the cointegrating rank are available in the *Initial Analysis* part of JMULTi. The  $\Gamma_i$ ,  $B_j$  and  $C$  are also parameter matrices with suitable dimensions.

Various restrictions can be imposed on the parameter matrices. In particular, it is necessary to impose restrictions to ensure an identified model form which can be estimated. Generally, (1) is a structural form which can only be estimated if identifying restrictions are imposed. If  $\Gamma_0$  is specified to be an identity matrix, the model becomes a reduced form.

By imposing zero restrictions, the right-hand side variables may not be the same in all equations. For example, some equations may contain specific dummy or exogenous variables which do not appear in other equations. Notice also that  $B_0 = 0$  may be specified if the exogenous variables are to appear in lagged form only. It is also possible to specify a model

without any exogenous variables.

The number of lagged differences of the endogenous variables,  $p$ , may be chosen with the help of model selection criteria (see Sec. 3.1).

It may also be worth noting that the model (1) can be rearranged such that all endogenous variables appear in levels only and the deterministic terms are all collected in the vector  $D_t^*$ :

$$A_0 y_t = A_1 y_{t-1} + \dots + A_{p+1} y_{t-p-1} + B_0 x_t + \dots + B_q x_{t-q} + C^* D_t^* + u_t. \quad (2)$$

This model is similar but not identical to the most general VAR model available in JMulTi. In the VAR part,  $A_0$  is restricted to be an identity matrix, so that the VARs are in reduced form. Notice, that  $p + 1$  lags of  $y_t$  appear on the right-hand side of (2) while there are only  $p$  lags of  $\Delta y_t$  in (1).

## 1.1 General Remark about the Implementation of VECMs in JMulTi

VEC modelling in JMulTi is a step by step procedure, where each task is related to a special panel. Once a model has been estimated, the diagnostic tests as well as the stability analysis, structural analysis and forecasting use the results from the estimation. If changes in the model specification are made by the user, these results are deleted and the model has to be reestimated. In other words, only results related to one model at a time are kept in the system. Hence, there should be no confusion regarding the model setup while going through the analysis.

Sometimes certain menus or options are not available. This always has a specific reason which is described in the respective help topic.

## 2 Estimation

### 2.1 Background

There are different estimation procedures available for estimating a model of the type (1), depending on the precise model specification. If  $\Gamma_0 = I$ , there are no zero restrictions on the  $\Gamma_j$  matrices ( $j = 1, \dots, p$ ) and there are no exogenous variables, that is, a reduced form model is specified without exogenous variables and where each equation has the same right-hand side variables, then the Johansen reduced rank (RR) estimation procedure (see Johansen (1995)) and a simple two step (S2S) method (see Ahn and Reinsel (1990) or Lütkepohl and Krätzig (2004, Chapter 3)) can be applied. If a structural form with  $\Gamma_0 \neq I$  is specified or there are restrictions on the  $\Gamma_j$  ( $j = 1, \dots, p$ ) or there are exogenous variables, then a two stage estimation procedure can be used in JMulTi. There are different options for the first and second step of the two stage procedure. The different estimation methods are briefly described in the following.

#### 2.1.1 Johansen Procedure

The Johansen RR procedure can be used to estimate models of the form

$$\Delta y_t = \alpha \beta^{*t} \begin{bmatrix} y_{t-1} \\ D_{t-1}^{co} \end{bmatrix} + \Gamma_1 \Delta y_{t-1} + \dots + \Gamma_p \Delta y_{t-p} + CD_t + u_t$$

without parameter restrictions (see Johansen (1995) for details). The cointegration matrix  $\beta^*$  is automatically normalized as follows:

$$\beta^* = \begin{bmatrix} I_r \\ \beta_{(K^*-r)}^* \end{bmatrix},$$

where  $\beta_{(K^*-r)}^*$  is a  $((K^* - r) \times r)$  matrix. This normalization requires that the order of the variables is specified such that the first  $r$  variables are actually involved in the cointegration relations. In other words, meaningful cointegration relations must result with the normalization.

If the Johansen procedure is used for estimation, there is also a possibility to test restrictions of the form

$$H_0 : R\text{vec}(\beta_{(K^*-r)}^*) = \mathbf{r} \quad \text{versus} \quad H_1 : R\text{vec}(\beta_{(K^*-r)}^*) \neq \mathbf{r}.$$

If there are  $J$  linearly independent restrictions for the coefficients of  $\beta_{(K^*-r)}^*$ , then  $R$  is a  $(J \times (K^* - r)r)$  matrix and  $\mathbf{r}$  is a  $J$ -dimensional vector. A Wald test is used with a  $\chi^2(J)$  distribution.

Although restrictions on the cointegration relations can be tested on the basis of the Johansen estimators, JMULTi does not offer the possibility to impose the restrictions in the RR estimation procedure.

### 2.1.2 S2S Estimation

The simple two step (S2S) estimator is also only applicable for reduced form models ( $\Gamma_0 = 0$ ) without exogenous variables and without restrictions on the short-term dynamics. If the latter condition holds, we can concentrate out the short-term dynamics and deterministic terms outside the cointegration relations ( $CD_t$ ) by replacing the parameters by their LS estimators conditional on the parameters in the error correction term. This can be done by regressing  $\Delta y_t$  and

$$\begin{bmatrix} y_{t-1} \\ D_{t-1}^{co} \end{bmatrix}$$

on the short-term dynamics and  $CD_t$ . Denoting the residuals by  $\Delta \tilde{y}_t$  and  $\tilde{y}_{t-1}^*$  and using the normalization

$$\beta^* = \begin{bmatrix} I_r \\ \beta_{(K^*-r)}^* \end{bmatrix},$$

we can write

$$\Delta \tilde{y}_t - \alpha \tilde{y}_{t-1}^{(1)} = \alpha \beta_{(K^*-r)}^{*'} \tilde{y}_{t-1}^{(2)} + \tilde{u}_t,$$

where  $\tilde{y}_{t-1}^{(1)}$  consists of the first  $r$  components of  $\tilde{y}_{t-1}^*$  and  $\tilde{y}_{t-1}^{(2)}$  contains the last  $K^* - r$  components of  $\tilde{y}_{t-1}^*$ . Premultiplying by  $(\alpha' \Sigma_u^{-1} \alpha)^{-1} \alpha' \Sigma_u^{-1}$  and defining

$$w_t = (\alpha' \Sigma_u^{-1} \alpha)^{-1} \alpha' \Sigma_u^{-1} (\Delta \tilde{y}_t - \alpha \tilde{y}_{t-1}^{(1)}),$$

gives

$$w_t = \beta_{(K^*-r)}^{*'} \tilde{y}_{t-1}^{(2)} + v_t, \tag{3}$$

where  $v_t = (\alpha' \Sigma_u^{-1} \alpha)^{-1} \alpha' \Sigma_u^{-1} \tilde{u}_t$ .

If  $\alpha$  and  $\Sigma_u$  were given,  $\beta_{(K^*-r)}^{*'}$  could be estimated by LS from this model. Because the former parameters are unknown in practice, the following 2-step procedure may be used. In the first step  $\Pi = [\alpha : \alpha \beta_{(K^*-r)}^{*'}]$  is estimated by LS from the concentrated model

$$\Delta \tilde{y}_t = \Pi \tilde{y}_{t-1}^* + \tilde{u}_t.$$

The first  $r$  columns of the estimated  $\Pi$  are used as an estimator of  $\alpha$  and the residual covariance estimator is used for  $\Sigma_u$ . These quantities are substituted in the definition of  $w_t$  and  $\beta_{(K^*-r)}^{*'}$  is estimated by LS from the resulting multivariate model corresponding to (3). Denoting the estimator by  $\tilde{\beta}_{(K^*-r)}^{*'}$  and defining

$$\tilde{\beta}^* = \begin{bmatrix} I_r \\ \tilde{\beta}_{(K^*-r)}^{*' \prime} \end{bmatrix},$$

the remaining parameters are estimated by LS from the model

$$\Delta y_t = \alpha \widetilde{\beta}^{*'} \begin{bmatrix} y_{t-1} \\ D_{t-1}^{co} \end{bmatrix} + \Gamma_1 \Delta y_{t-1} + \dots + \Gamma_p \Delta y_{t-p} + CD_t + \hat{u}_t.$$

Linear restrictions on the cointegration matrix of the form

$$\text{vec}(\beta_{(K^*-r)}^{*'}) = \mathcal{H}\eta + h$$

can be accommodated in the S2S estimation (see Lütkepohl and Krätzig (2004, Chapter 3)). Here  $\mathcal{H}$  is a fixed matrix,  $h$  a fixed vector and  $\eta$  a vector of free parameters. These restrictions may be formulated alternatively as

$$R\text{vec}(\beta_{(K^*-r)}^{*'}) = \mathbf{r},$$

where  $R$  is a  $(J \times (K^* - r)r)$  matrix and  $\mathbf{r}$  is a  $J$ -dimensional vector, as before.

### 2.1.3 Two Stage Estimation

In the two stage procedure restrictions can also be imposed on the loading coefficients, the short-term dynamics and the deterministic terms. Moreover, there may be exogenous variables and structural restrictions. In other words the general from (1) can be estimated with the two stage procedure.

In the first stage the cointegration matrix  $\beta^*$  has to be estimated or specified. If the user knows the cointegration relations, they may be specified directly. If there is just one cointegrating relation it may be estimated from a single equation of the VECM. The user may specify the equation to be used. The equation is then estimated by OLS and the cointegration relation is extracted by normalizing the coefficient of the first variable to 1 (see, e.g., Lütkepohl (1991, Chapter 11)). Alternatively, the cointegration matrix may be estimated by Johansen's RR procedure or by the S2S method. Note, however, that in **JMulTi** all exogenous variables have to be eliminated from the model for performing this step. Also the model must be set up in reduced form and subset restrictions cannot be imposed in the latter two methods. The cointegration matrix obtained from the RR regression will automatically be normalized such the the first part is an identity matrix. Thereby an identified form of the estimated  $\beta^*$  matrix is available for both the RR and the S2S method which can be used in the second stage of the estimation procedure.

In the second stage, structural and subset restrictions as well as exogenous variables can be accounted for. The term  $\hat{\beta}^{*'} y_{t-1}^*$  is treated as an additional set of variables. The user may specify the estimation strategy to be used in the second stage. If he/she wants **JMulTi** to make a choice, the **Automatic** option can be selected. In that case, OLS for each individual

equation will be used if the set of regressors is the same in each of the equations. Feasible systems GLS will be used if the model is in reduced form and contains subset restrictions. In this procedure, the individual equations of the system are first estimated by OLS. The residuals are used to estimate the white noise covariance matrix  $\Sigma_u$  as  $\hat{\Sigma}_u = T^{-1} \sum_{t=1}^T \hat{u}_t \hat{u}_t'$  and this estimator is then used in the next step to compute the GLS estimator. If the VECM is in structural form with  $\Gamma_0 \neq I$ , a 3SLS (three stage least squares) procedure is used. JMulTi does a check of the identification of the model. No estimation can be performed if JMulTi is not sure about the identification, based on an order criterion. In all of these procedures, the cointegration relations estimated or specified in the first stage are included as additional regressor variables. The user also has the option of selecting OLS, GLS or 3SLS rather than leaving the choice to JMulTi. This makes sense, for example, if an OLS rather than GLS procedure is desired for a reduced form with zero restrictions on some of the coefficient matrices.

## 2.2 Estimation Options in JMulTi

On the right-hand side of the specification panel for VECMs the variables must be selected and the sample period has to be specified. Then the number of lagged differences of the endogenous variables, the number of lags of the exogenous variables - if applicable -, the cointegration rank and the deterministic terms to be included can be specified in the upper left hand part of the panel. See Sec. 3.2 for details.

The estimation method has to be specified in the lower left-hand part of the specification panel. To impose restrictions on the cointegration relations, choose **Specification**  $\rightarrow$  **Specify and Test Restrictions on Beta**. A menu appears which allows you to specify the restrictions. They will only be taken into account if the S2S estimation procedure is selected in the VECM specification menu. If restrictions are specified, the **Delete Restrictions** button can be used to deactivate them. If the **Two stage procedure** is selected, a structural form may be specified by checking the corresponding box and pressing the **Structural Form** button. Then a menu appears which allows to specify elements in the  $\Gamma_0$  matrix which are not restricted to be zero. Note that the diagonal elements of the matrix have to be ones. Moreover, there must be enough zeros to ensure identification. Also, in the **Two stage procedure** the first and second stages have to be specified as described in Sec. 2.1.3.

## 2.3 Estimation Results in JMulTi

After the model and the estimation procedure are specified properly, see Sec. 3, it can be estimated by selecting the menu **Estimation** $\rightarrow$ **Estimate Model**. Output is generated in matrix

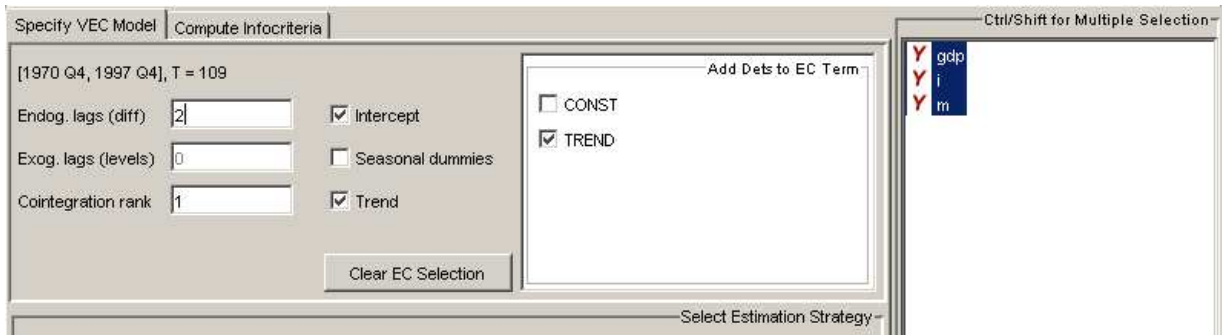


Figure 1: Specification of Estimation Strategy for VECM

and text form.

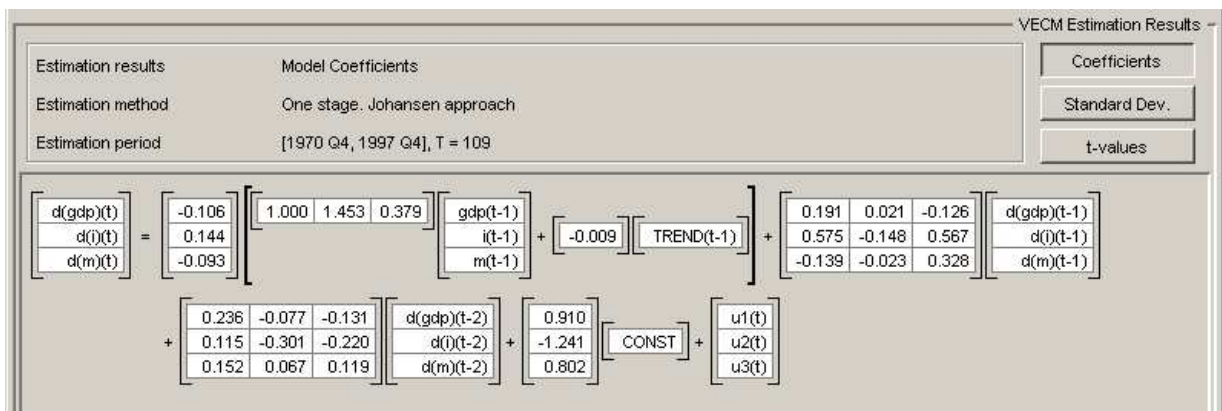


Figure 2: Estimation Results in Matrix (VEC) Form

The Matrix(VEC) panel displays the estimated parameters of the VEC form of the model, starting with the endogenous, then exogenous and finally the parameter matrices associated with deterministic terms. This arrangement reflects the mathematical notation to make clear, what type of model was actually estimated. By RIGHT clicking on the coefficients tables one can increase or decrease the precision of the numbers. By clicking on the respective buttons it is possible to display either the estimated coefficients, the standard deviations or the  $t$ -values.

There is also an option to display the corresponding levels form (2) of the model. In this from only the estimated coefficients and no standard errors or  $t$ -ratios are provided.

The text panel displays the same information as the two matrix panels but in a form that can be saved as a text file by RIGHT clicking over the text area. In addition to that, it gives



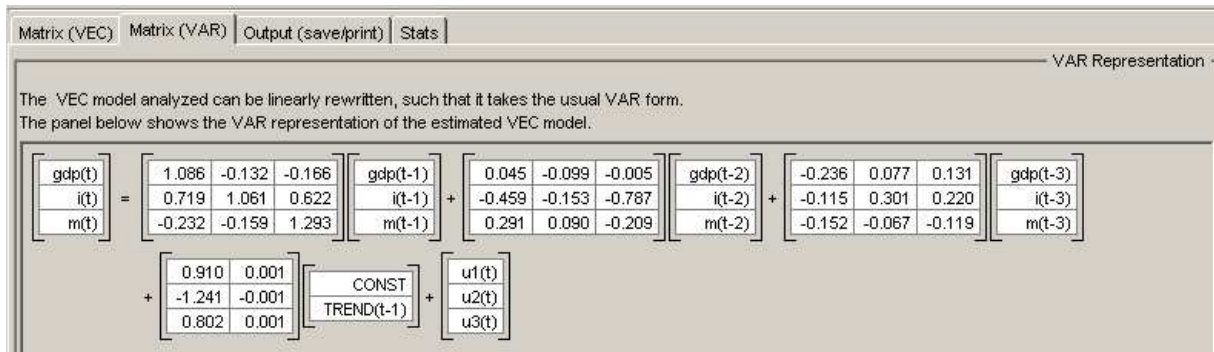


Figure 3: Estimation Results in Matrix (VAR) Form

the modulus of the eigenvalues of the reverse characteristic polynomial, which is defined as:

$$\det(I_K - A_1 z - \dots - A_p z^p), \quad \text{see Lütkepohl (1991), Ch. 2.}$$

The Stats panel shows the sample range, the Gaussian log likelihood value corresponding to the estimates, the residual covariance matrix and its determinant as well as the corresponding correlation matrix.

Matrix (VEC)	Matrix (VAR)	Output (save/print)	Stats
...			
-----			
Lagged endogenous term:			
=====			
	d(gdp)	d(i)	d(m)
-----			
d(gdp) (t-1)	0.191	0.575	-0.139
	(0.087)	(0.126)	(0.070)
	{0.028}	{0.000}	{0.048}
	[2.197]	[4.551]	[-1.977]
d(i) (t-1)	0.021	-0.148	-0.023
	(0.087)	(0.127)	(0.071)
	{0.806}	{0.242}	{0.746}
	[0.245]	[-1.171]	[-0.323]
d(m) (t-1)	-0.126	0.567	0.328
	(0.120)	(0.174)	(0.097)
	{0.293}	{0.001}	{0.001}
	[-1.051]	[3.264]	[3.385]
d(gdp) (t-2)	0.236	0.115	0.152
	(0.094)	(0.136)	(0.076)
	{0.012}	{0.397}	{0.045}
	[2.512]	[0.847]	[2.005]
d(i) (t-2)	-0.077	-0.301	0.067

Figure 4: Estimation Results in Text Form

Matrix (VEC)	Matrix (VAR)	Output (save/print)	Stats
*****			
VECM MODEL STATISTICS			
sample range: [1970 Q4, 1997 Q4], T = 109			
Log Likelihood:	1.149998e+03		
Determinant (Cov):	1.131703e-13		
Covariance:	4.626741e-05	1.358276e-05	-2.449571e-06
	1.358276e-05	9.713925e-05	-1.949244e-05
	-2.449571e-06	-1.949244e-05	3.017146e-05
Correlation:	1.000000e+00	2.026063e-01	-6.556232e-02
	2.026063e-01	1.000000e+00	-3.600566e-01
	-6.556232e-02	-3.600566e-01	1.000000e+00

Figure 5: Some Statistics Related to Estimation Results

### 3 Model Specification

Specifying a model of the form (1) in JMulTi involves the specification of the cointegrating rank as well as a maximum lag order for the endogenous and exogenous variables. Moreover, zero restrictions may be placed on the parameter matrices. The cointegrating rank can be specified with the help of the cointegration tests available in the **Initial Analysis** part. In the **VECM** part model selection criteria are available to aid in the choice of the number  $p$  of lagged differences of the endogenous variables. Moreover different procedures for imposing zero restrictions on the parameter matrices are offered (see Sec. 4).

#### 3.1 Model Selection Criteria

The information criteria are computed for reduced form VECMs

$$\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \cdots + \Gamma_n \Delta y_{t-n} + B_0 x_t + \cdots + B_q x_{t-q} + C^* D_t^* + u_t,$$

where  $y_t$  is  $K$ -dimensional. Note that here the matrix  $\Pi$  replaces the term  $\alpha\beta'$  in the model (1). Its rank is left unrestricted so that the estimated matrix will have rank  $K$ . The lag order  $q$  of the exogenous variables  $x_t$  and the deterministic term  $D_t^*$  have to be prespecified by the analyst. For a range of lag orders  $n$  the model is estimated by OLS (applied to each equation separately). The optimal lag order is chosen by minimizing one of the usual information criteria:

$$\begin{aligned} \text{AIC}(n) &= \log \det(\tilde{\Sigma}_u(n)) + \frac{2}{T} n K^2, \\ \text{HQ}(n) &= \log \det(\tilde{\Sigma}_u(n)) + \frac{2 \log \log T}{T} n K^2, \\ \text{SC}(n) &= \log \det(\tilde{\Sigma}_u(n)) + \frac{\log T}{T} n K^2 \end{aligned}$$

and

$$\text{FPE}(n) = \left( \frac{T + n^*}{T - n^*} \right)^K \det(\tilde{\Sigma}_u(n)),$$

where  $\tilde{\Sigma}_u(n)$  is estimated by  $T^{-1} \sum_{t=1}^T \hat{u}_t \hat{u}_t'$ ,  $\hat{u}_t$  being the estimation residual vectors. The quantity  $n^*$  is the total number of parameters in each equation of the model when  $n$  is the lag order of the endogenous variables, also counting the levels variables, the deterministic terms and exogenous variables. The sample length is the same for all different lag lengths and is determined by the maximum lag order. In other words, the number of values set aside as presample values is determined by the maximum lag order considered for the endogenous and exogenous variables. The lag length which minimizes the respective information criterion is presented in the output of JMulTi. See also the Model Selection part for VAR models.

The model selection criteria can be accessed via **Specification** → **Specify VEC Model** → **Compute Infocriteria**. Then a menu appears where the maximum lag length can be specified. The optimal lag order is computed upon pressing the button **Compute Infocriteria**.

### 3.2 Selection of Variables, Lags and Cointegrating Rank in JMulTi

To get to the specification panel you need to select **Specification**→**Specify VEC Model**. For building a model of the form (1) in JMulTi you have to choose the variables you want to include in your model first. By **RIGHT** clicking on the selected variables you may define exogenous or deterministic variables. The selected user defined variables are shown in their correct order in the available text fields.

You may also adjust the sample by editing the date text fields in the selection panel. To include intercept, trend or seasonal dummies, you should use the available checkboxes. But it is possible to add further deterministic variables defined by the user.

For selecting the lags for the differenced endogenous variables it may be helpful to use the information criteria. To do that, a model with the selected variables is estimated. The exogenous lags are taken as given. A search is performed over the lags of the endogenous variables up to the maximum order.

The cointegrating rank  $r$  must be specified in the range  $1, \dots, K - 1$ .

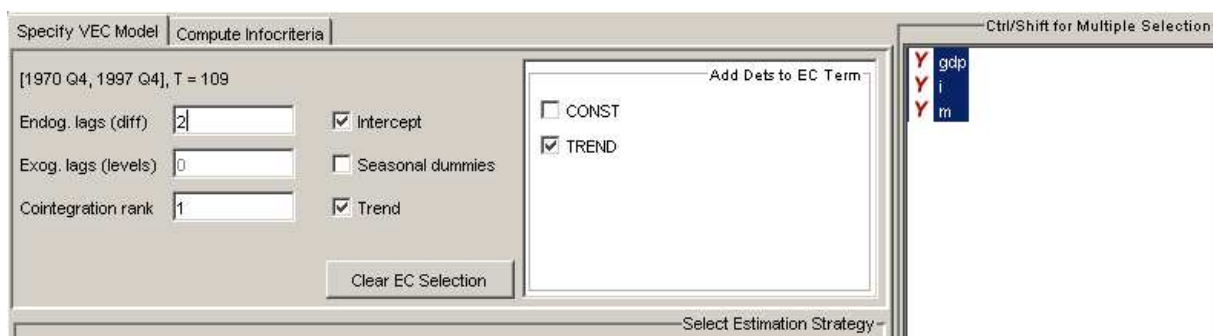


Figure 6: Specification of Variables, Sample Period, Lags and Cointegrating Rank

### 3.3 Specification of Deterministic Terms

If deterministic terms are to be included in the cointegration relations, this may be accomplished in the **Specify VEC Model** menu by checking the relevant terms in the **Add Dets to EC Term** panel. For a particular model setup, only the terms offered in the panel are possible choices.

### 3.4 Specification of Structural Form Parameters

VECMs can be specified in structural form with instantaneous relations between the endogenous variables via proper specification of  $\Gamma_0$  in (1). If this matrix is not specified, JMulTi will automatically choose an identity matrix and set up the model in reduced form. Other  $\Gamma_0$  matrices can be specified by checking **Estimate structural form** in the **Specify VEC Model** panel. This is only possible if the two stage estimation procedure was selected previously. In that case a menu opens up which allows to specify off-diagonal elements to be estimated unrestrictedly. JMulTi will check identification problems by checking the order condition for identification and informing the user if the condition is not satisfied.

## 4 Subset Model Selection in VECMs

### 4.1 Background

Zero restrictions may be imposed on the parameters of a model based on the  $t$ -ratios, for example. Alternatively, restrictions for individual parameters or groups of parameters may be based on model selection criteria. **JMulTi** offers suitable model selection procedures based on single equation methods as well as algorithms which consider the full system at once in the elimination procedure.

To describe the single equation methods consider the equation

$$y_t = \theta_1 x_{1t} + \cdots + \theta_N x_{Nt} + u_t, \quad t = 1, \dots, T. \quad (4)$$

For simplicity, all right-hand side variables are denoted by  $x_{kt}$  including estimated cointegration relations which are added as additional variables, exogenous and deterministic variables as well as lagged endogenous variables. The optimal set of regressors is then selected by minimizing a variable selection criterion of the general form

$$CR(i_1, \dots, i_n) = \log(SSE(i_1, \dots, i_n)/T) + c_T n/T, \quad (5)$$

where  $SSE(i_1, \dots, i_n)$  is the sum of squared errors obtained by including  $x_{i_1 t}, \dots, x_{i_n t}$  in the regression model (4) and  $c_T$  is a quantity which determines the specific criterion. More precisely,

$$c_T = \begin{cases} 2 & \text{for AIC,} \\ 2 \log \log T & \text{for HQ,} \\ \log T & \text{for SC.} \end{cases}$$

The following **Sequential Elimination of Regressors (SER)** strategy is available in **JMulTi**: Sequentially delete those regressors which lead to the largest reduction of the given criterion until no further reduction is possible (see, e.g., Brüggemann and Lütkepohl (2001) for more details). This strategy is equivalent to sequentially eliminating those regressors with the smallest absolute values of  $t$ -ratios until all  $t$ -ratios (in absolute value) are greater than some threshold value. Note that a single regressor is eliminated in each step only. Then new  $t$ -ratios are computed for the reduced model.

Another possible sequential elimination algorithm implemented in **JMulTi** is a **Top-Down (TD)** procedure which starts from the last regressor in the equation and checks if deleting it reduces the criterion value. In that case it is eliminated. Otherwise it is maintained. Then the second last regressor is checked and so on. Obviously, this procedure depends on the ordering of the variables in the model and, hence, in the equation.

There is also a **System SER** procedure implemented in **JMulTi**. In this procedure, in each step the parameter with the smallest  $t$ -ratio is checked and potentially eliminated. The decision regarding the elimination can be based on model selection criteria or a threshold value is specified and only variables with a  $t$ -ratio larger than the threshold are maintained eventually.

Because identification problems may arise in the course of a selection run in structural from models with  $\Gamma_0 \neq I$ , the subset selection procedures only work for reduced form models. Therefore, to apply the automatic subset selection procedures, all nonzero off-diagonal elements of  $\Gamma_0$  have to be replaced by zero.

## 4.2 Specification of Subset Restrictions and Search Strategy in JMulTi

To get to the subset specification panel you need to select **Specification**→**Specify Subset Model**. There you see the full model defined in the selection panel, see Sec. 3, with the possibility to include or exclude certain coefficients from the estimation. You may use the **RIGHT** mouse click over the tables to set a property for whole matrices.

It is also possible to let **JMulTi** search for restrictions automatically. To do this you need to define a search strategy. The following strategies are available:

**System SER** see 4.1, selection based on information criterion

**SER/Testing Procedure** see 4.1

**Top Down** see 4.1

**System Testing Procedure** see 4.1, selection based on a threshold value

For each strategy one may select the model selection criterion, for the **System Testing Procedure** the threshold value can be specified. You may exclude or include certain variables regardless of what the search procedure tells you by setting them either to **0** or to **!**. If you do not use a search procedure but estimate the model with manually set restrictions, there is no difference between **!** and **\***. The display of the results of the estimation with subset restrictions is described in Sec. 2.3.

Specify Subset Restrictions

System SER       AIC      Click on coefficients:  
 SER / Testing Procedure       HQ      0 : Excludes from model  
 Top Down       SC      ^ : Includes in model, searched  
 System Testing Procedure       Threshold      ! : Includes in model, not searched

Please note: Subset restrictions are only taken into account if TWO STAGE estimation is selected (current: JOHANSEN).

$$\begin{bmatrix} d(gdp)(t) \\ d(i)(t) \\ d(m)(t) \end{bmatrix} = \begin{bmatrix} * \\ * \\ * \end{bmatrix} \begin{bmatrix} ec1(t-1) \end{bmatrix} + \begin{bmatrix} * & * & * \\ * & * & 0 \\ * & * & * \end{bmatrix} \begin{bmatrix} d(gdp)(t-1) \\ d(i)(t-1) \\ d(m)(t-1) \end{bmatrix} + \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & 0 \end{bmatrix} \begin{bmatrix} d(gdp)(t-2) \\ d(i)(t-2) \\ d(m)(t-2) \end{bmatrix} + \begin{bmatrix} * \\ * \\ * \end{bmatrix} \begin{bmatrix} CONST \end{bmatrix}$$

Figure 7: Specification of Subset Restrictions and Search Strategy for VECMs



## 5 Plotting the EC term

It is possible to plot and display different representations of the  $r$  estimated error correction terms. The following representation are available:

- $\beta_K^* y_{t-1}$
- $\beta^{*'} \begin{bmatrix} y_{t-1} \\ D_{t-1}^{co} \end{bmatrix}$
- $\beta_K^* y_{t-1} M$
- $\beta^{*'} \begin{bmatrix} y_{t-1} \\ D_{t-1}^{co} \end{bmatrix} M$

with  $\beta_K^*$  denoting the first  $K$  rows of  $\beta^*$  that correspond to the  $K$  lagged endogenous variables in the cointegration relation. The matrix  $M$  is defined as

$$M = I_T - X(X'X)^{-1}X'$$

with  $X' = [X_0, \dots, X_{T-1}]$ ,  $X'_{t-1} = (\Delta y'_{t-1}, \dots, \Delta y'_{t-p}, CD'_t, x', x'_{t-1}, \dots, x'_{t-q})$  Multiplication with  $M$  has the effect of eliminating the short-run dynamics.

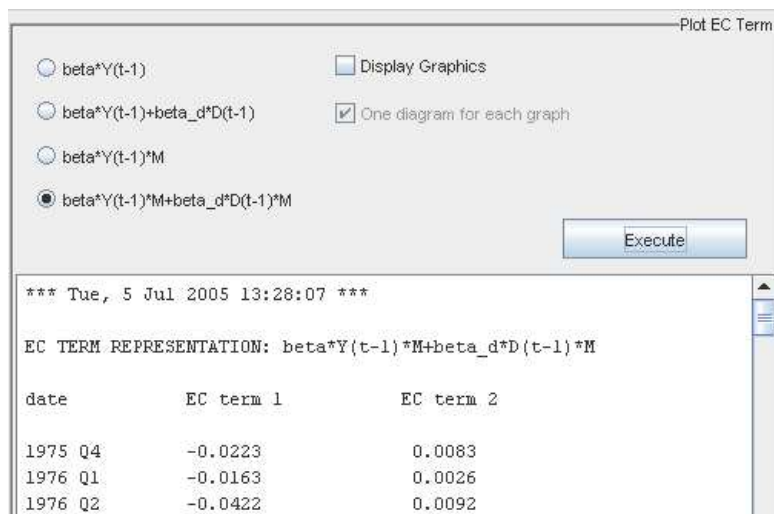


Figure 8: EC plot panel

## 6 Residual Analysis

To access the residual analysis you have to specify and estimate a model first. It is then possible to select the menu item **Model Checking**→**Residual Analysis**. In the following the estimation residuals or residual vectors are denoted by  $\hat{u}_t$ . In JMulTi the residual analysis is split up into several different panels.

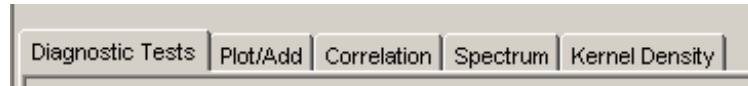


Figure 9: Available Panels for Residual Analysis

**Diagnostic Tests** to convey a range of diagnostic tests, see Sec. 7

**Plot/Add** residuals can be plotted in several ways as well as added again to the set of available series

**Correlation** for graphical autocorrelation and crosscorrelation analysis see Sec. 8

**Spectrum** it is possible to show the spectrum of the available residuals, see the respective description in helpsection **Initial Analysis**

**Kernel Density** for a description of kernel density estimation, see the respective description in helpsection **Initial Analysis**

## 7 Diagnostic Tests

### 7.1 Background

In JMulTi, tests for residual autocorrelation, nonnormality and conditional heteroskedasticity are available for checking the adequacy of estimated VECMs.

#### Portmanteau test for autocorrelation

A portmanteau test for residual autocorrelation may be applied if a pure VEC model possibly with subset restrictions but without exogenous variables has been fitted. The test checks the null hypothesis

$$H_0 : \mathbb{E}(u_t u'_{t-i}) = 0, \quad i = 1, \dots, h,$$

against the alternative that at least one autocovariance and, hence, the corresponding autocorrelation is nonzero. The test statistic has the form

$$Q_h = T \sum_{j=1}^h \text{tr}(\hat{C}'_j \hat{C}_0^{-1} \hat{C}_j \hat{C}_0^{-1})$$

where  $\hat{C}_i = T^{-1} \sum_{t=i+1}^T \hat{u}_t \hat{u}'_{t-i}$ . The test results in JMulTi use an approximation to the distribution of the test statistic under the null hypothesis based on a  $\chi^2(K^2 h - n^*)$  distribution. Here  $n^*$  denotes the number of estimated loading ( $\alpha$ ) and short-run parameters ( $\Gamma_j$ ) in the VEC model under consideration. Restricted parameters do not count and also the deterministic terms do not count in evaluating  $n^*$ . The following adjusted portmanteau statistic is also available,

$$Q_h^* = T^2 \sum_{j=1}^h \frac{1}{T-j} \text{tr}(\hat{C}'_j \hat{C}_0^{-1} \hat{C}_j \hat{C}_0^{-1}).$$

Its distribution may be closer to the approximating  $\chi^2$  distribution under the null hypothesis. The choice of  $h$  is important for the test performance. If  $h$  is chosen too small, the  $\chi^2$  approximation to the null distribution may be very poor whereas a large  $h$  may result in a loss of power.

#### LM tests for autocorrelation

The LM test for  $h$ -th order residual autocorrelation assumes a model

$$u_t = B_1^* u_{t-1} + \dots + B_h^* u_{t-h} + \text{error}_t$$

and checks

$$H_0 : B_1^* = \dots = B_h^* = 0 \quad \text{vs.} \quad H_1 : B_1^* \neq 0 \text{ or } \dots \text{ or } B_h^* \neq 0.$$

The auxiliary model

$$\begin{aligned} \Gamma_0 \hat{u}_t = & \alpha \beta^{*'} \begin{bmatrix} y_{t-1} \\ D_{t-1}^{co} \end{bmatrix} + \Gamma_1 \Delta y_{t-1} + \cdots + \Gamma_p \Delta y_{t-p} \\ & + B_0 x_t + \cdots + B_q x_{t-q} + CD_t + B_1^* \hat{u}_{t-1} + \cdots + B_h^* \hat{u}_{t-h} + e_t \end{aligned}$$

is considered. The model is estimated by the same method as the original model with  $\hat{u}_t$ ,  $t \leq 0$ , replaced by zero.

Denoting the estimation residuals by  $\hat{e}_t$  ( $t = 1, \dots, T$ ), the residual covariance matrix estimator obtained from the auxiliary models is

$$\tilde{\Sigma}_e = \frac{1}{T} \sum_{t=1}^T \hat{e}_t \hat{e}_t'$$

Moreover, reestimating the relevant auxiliary model without the lagged residuals  $\hat{u}_{t-i}$ , that is, imposing the restriction  $B_1^* = \cdots = B_h^* = 0$ , and denoting the resulting residuals by  $\hat{e}_t^R$ , the corresponding covariance matrix estimator is

$$\tilde{\Sigma}_R = \frac{1}{T} \sum_{t=1}^T \hat{e}_t^R \hat{e}_t^{R'}$$

The LM statistic is

$$LM_h = T \left( K - \text{tr}(\tilde{\Sigma}_R^{-1} \tilde{\Sigma}_e) \right) \approx \chi^2(hK^2).$$

### Tests for nonnormality

The idea underlying the nonnormality tests is to transform the residual vector such that its components are independent and then check the compatibility of the third and fourth moments with those of a normal distribution. In a first step, the residual covariance matrix is estimated as

$$\tilde{\Sigma}_u = T^{-1} \sum_{t=1}^T (\hat{u}_t - \bar{\hat{u}})(\hat{u}_t - \bar{\hat{u}})'$$

and the square root matrix  $\tilde{\Sigma}_u^{1/2}$  is computed. The tests for nonnormality may be based on the skewness and kurtosis of the standardized residuals  $\hat{u}_t^s = (\hat{u}_{1t}^s, \dots, \hat{u}_{Kt}^s)' = \tilde{\Sigma}_u^{1/2}(\hat{u}_t - \bar{\hat{u}})$ :

$$\mathbf{b}_1 = (b_{11}, \dots, b_{1K})' \quad \text{with} \quad b_{1k} = T^{-1} \sum_{t=1}^T (\hat{u}_{kt}^s)^3$$

and

$$\mathbf{b}_2 = (b_{21}, \dots, b_{2K})' \quad \text{with} \quad b_{2k} = T^{-1} \sum_{t=1}^T (\hat{u}_{kt}^s)^4.$$

Defining

$$s_3^2 = T \mathbf{b}_1' \mathbf{b}_1 / 6$$

and

$$s_4^2 = T(\mathbf{b}_2 - \mathbf{3}_K)'(\mathbf{b}_2 - \mathbf{3}_K)/24,$$

where  $\mathbf{3}_K = (3, \dots, 3)'$  is a  $(K \times 1)$  vector, a multivariate version of a Lomnicki-Jarque-Bera statistic is

$$LJB_K = s_3^2 + s_4^2.$$

The statistics  $s_3^2$  and  $s_4^2$  have  $\chi^2(K)$  limiting distributions and  $JB_K$  has a  $\chi^2(2K)$  asymptotic distribution if the normality null hypothesis holds. The latter statistic was proposed by Doornik and Hansen (1994).

An alternative way of computing standardized residuals was considered by Lütkepohl (1991, Chapter 4) who uses a Choleski decomposition of the residual covariance matrix. Let  $\tilde{P}$  be a lower triangular matrix with positive diagonal such that  $\tilde{P}\tilde{P}' = \tilde{\Sigma}_u$ . Then the residuals are standardized as  $\hat{u}_t^s = \tilde{P}^{-1}(\hat{u}_t - \bar{u})$ . Computing the third and fourth moments as in the foregoing as well as  $s_{3L}^2$  and  $s_{4L}^2$  corresponding to  $s_3^2$  and  $s_4^2$ , respectively, gives  $LJB_K^L = s_{3L}^2 + s_{4L}^2$  with asymptotic  $\chi^2(2K)$  distribution under normality. Again,  $s_{3L}^2$  and  $s_{4L}^2$  have  $\chi^2(K)$  limiting distributions.

### ARCH-LM test

A multivariate ARCH-LM test may be based on the multivariate regression model

$$\text{vech}(\hat{u}_t \hat{u}_t') = \beta_0 + B_1 \text{vech}(\hat{u}_{t-1} \hat{u}_{t-1}') + \dots + B_q \text{vech}(\hat{u}_{t-q} \hat{u}_{t-q}') + \text{error}_t, \quad (6)$$

where  $\text{vech}$  is the column stacking operator for symmetric matrices which stacks the columns from the main diagonal downwards,  $\beta_0$  is  $\frac{1}{2}K(K+1)$ -dimensional and the  $B_j$  are  $(\frac{1}{2}K(K+1) \times \frac{1}{2}K(K+1))$  coefficient matrices ( $j = 1, \dots, q$ ). The pair of hypotheses

$$H_0 : B_1 = \dots = B_q = 0 \quad \text{vs.} \quad H_1 : B_1 \neq 0 \text{ or } \dots \text{ or } B_q \neq 0$$

is tested. It is checked by the multivariate LM statistic

$$VARCH_{LM}(q) = \frac{1}{2}TK(K+1)R_m^2,$$

where

$$R_m^2 = 1 - \frac{2}{K(K+1)} \text{tr}(\hat{\Omega} \hat{\Omega}_0^{-1}),$$

$\hat{\Omega}$  is the residual covariance matrix of the  $\frac{1}{2}K(K+1)$ -dimensional regression model (6) and  $\hat{\Omega}_0$  is the corresponding matrix with  $q = 0$ . The statistic is similar to the one described by Doornik and Hendry (1997, Sec. 10.9.2.4) and the test is based on an approximate  $\chi^2(qK^2(K+1)^2/4)$  distribution. In addition univariate ARCH tests can be applied to the individual residual series. If ARCH of order  $q$  is tested, the univariate statistics may be used in conjunction with critical values or  $p$ -values based on a  $\chi^2(q)$  distribution. Alternatively, the statistic may be divided by  $q$  and used in conjunction with an  $F(q, T)$  distribution.

## 7.2 Implementation of Diagnostic Tests in JMulTi

The following tests are available:

**Portmanteau test** is only available for models without exogenous variables, see Sec. 7.1

**LM test** see Sec. 7.1

**Tests for nonnormality** multivariate and univariate versions are given, see Sec. 7.1

**ARCH-LM** multivariate and univariate versions can be selected, see Sec. 7.1

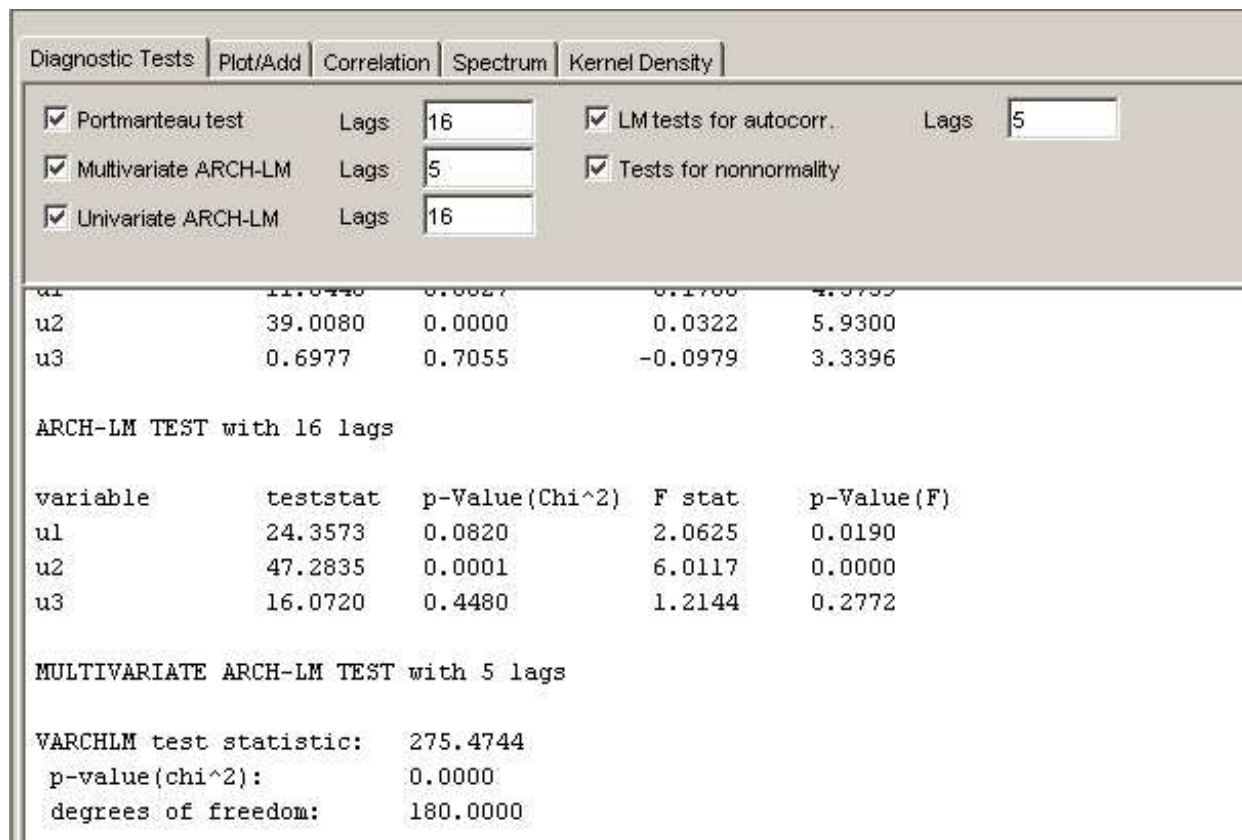


Figure 10: Diagnostic Tests for VEC Models in JMulTi

## 8 Correlation Analysis

### 8.1 Autocorrelation

In JMulTi residual autocorrelations (ACs) for a single residual series  $\hat{u}_t$ ,  $\tilde{\rho}_{u,h} = \tilde{\gamma}_{u,h}/\tilde{\gamma}_{u,0}$ , are obtained from

$$\tilde{\gamma}_{u,h} = \frac{1}{T} \sum_{t=h+1}^T (\hat{u}_t - \bar{\hat{u}})(\hat{u}_{t-h} - \bar{\hat{u}})$$

where  $\bar{\hat{u}} = T^{-1} \sum_{t=1}^T \hat{u}_t$  is the sample mean.

The partial autocorrelation (PAC) between  $u_t$  and  $u_{t-h}$  is the conditional autocorrelation given  $u_{t-1}, \dots, u_{t-h+1}$ . The corresponding sample quantity  $\hat{a}_h$  is obtained as the OLS estimator of the coefficient  $\alpha_h$  in an autoregressive model

$$\hat{u}_t = \nu + \alpha_1 \hat{u}_{t-1} + \dots + \alpha_h \hat{u}_{t-h} + error_t.$$

In JMulTi, OLS estimates are obtained for each  $h$  with sample size  $T - h$ . Approximate 95% confidence bounds specified as  $\pm 2/\sqrt{T}$  are used. ACs and PACs are computed for the single residual series.

### 8.2 Crosscorrelation

To get an overall picture of the correlation structure of the different residual series, a cross-correlation plot may be used. For VEC models the standard  $\pm 2/\sqrt{T}$  confidence bounds are provided in JMulTi.

### 8.3 Plotting Autocorrelations and Crosscorrelations in JMulTi

The available residuals are shown in the two lists from which the ones have to be selected for which autocorrelations or crosscorrelations are desired. The selection is done as usual by highlighting the residual series.

**plotting AC/PAC** select the desired series from the left list and press the **Autocorrelation** button

**plotting crosscorrelations** select the desired combination of series in the two lists and press the **Crosscorrelation** button

**Number of lags** the autocorrelation functions are computed up to the specified lag order

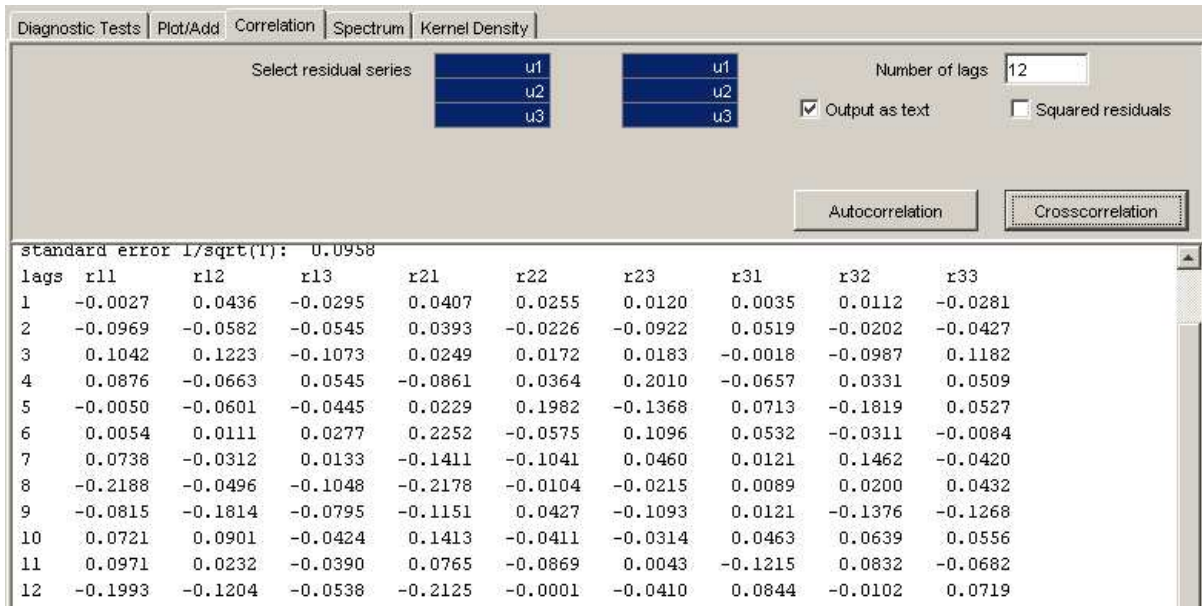


Figure 11: Correlation Analysis for VEC Models in JMulTi

**Squared residuals** the residuals are squared before the autocorrelations are computed



## 9 Stability Analysis

For VEC models, JMulTi offers the following options to check parameter constancy throughout the sample period: recursive estimates, recursive eigenvalues and Chow tests.

### 9.1 Recursive Coefficients

Recursive parameter estimates are obtained by simply estimating the model using only data for  $t = 1, \dots, \tau$  and letting  $\tau$  vary from some small value to  $T$ , the end of the original sample. Thereby sequences of coefficient estimates and estimates of the covariance matrix of the asymptotic distributions are obtained. The same estimation method is used that is also used for the full sample estimation. The cointegration relations are not reestimated, however. They are fixed at the full sample estimates in the recursive estimation. The series of estimates together with two-standard error bands are plotted and can convey useful information on the relative importance of new observations that are added to the sample.

#### Implementation in JMulTi

To get to the recursive parameter estimation panel you need to select **Model Checking** → **Stability Analysis** → **Recursive Coefficients**. The full model as specified in the selection panel is displayed for the user to choose the coefficients for which the graphs of recursive estimates are to be produced. Several choices have to be made before displaying the graphs.

**Start date** marks the beginning of the recursive estimation.

**Each coeff in separate window** displays each coefficient in a separate graph.

**Group different lags in separate windows** displays graphs with each containing all the coefficients of a parameter matrix as shown in the selection panel.

### 9.2 Recursive Eigenvalues

#### 9.2.1 Background

In JMulTi, for VEC models without parameter restrictions and without exogenous variables the eigenvalues from a reduced rank regression which are also used in the cointegration rank tests can be computed recursively by the Johansen estimation procedure (Johansen (1995)). Hansen and Johansen (1999) propose recursive statistics for stability analysis of VEC models which are partly available in JMulTi. Let  $\lambda_i^{(\tau)}$  be the  $i$ -th largest eigenvalue based on sample moments from the first  $\tau$  observations only. For a time invariant model, approximate 95%

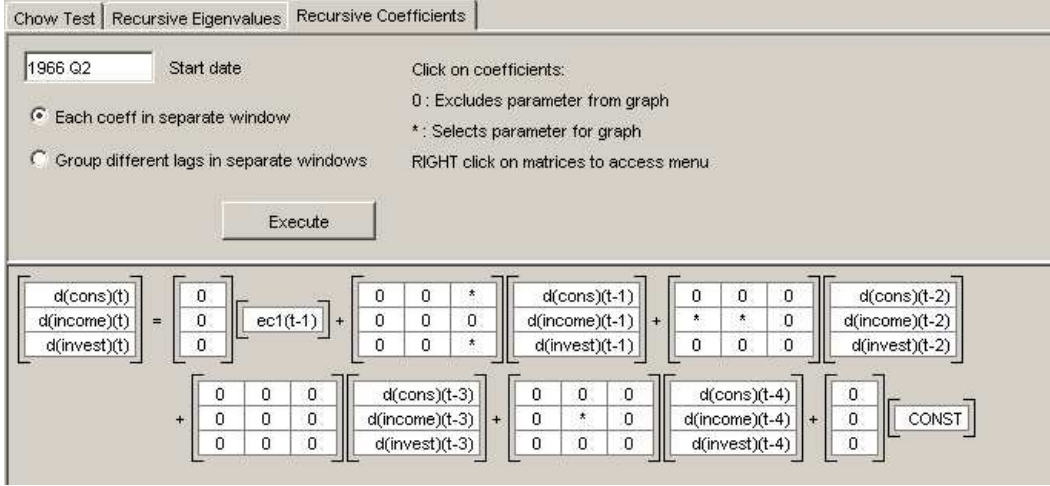


Figure 12: Recursive parameter estimates for VECM.

confidence intervals for the nonzero true eigenvalues corresponding to  $\lambda_1^{(\tau)}, \dots, \lambda_r^{(\tau)}$  are

$$\left[ \frac{\lambda_i^{(\tau)}}{\lambda_i^{(\tau)} + (1 - \lambda_i^{(\tau)}) \exp(1.96\hat{\sigma}_{ii})}, \frac{\lambda_i^{(\tau)}}{\lambda_i^{(\tau)} + (1 - \lambda_i^{(\tau)}) \exp(-1.96\hat{\sigma}_{ii})} \right].$$

Here  $\hat{\sigma}_{ii}^2$  is  $2/T$  times the  $i + (i - 1)[r + K(p - 1)]$ -th diagonal element of the matrix  $\Upsilon + \Upsilon'$  which in turn is based on

$$\Upsilon = (\tilde{\Sigma}^{-1} \otimes \tilde{\Sigma}^{-1})(I - \mathcal{A} \otimes \mathcal{A})^{-1},$$

where

$$\mathcal{A} = \begin{bmatrix} \tilde{\beta}'\tilde{\alpha} + I & \tilde{\beta}'\tilde{\Gamma}_1 & \cdots & \tilde{\beta}'\tilde{\Gamma}_{p-2} & \tilde{\beta}'\tilde{\Gamma}_{p-1} \\ \tilde{\alpha} & \tilde{\Gamma}_1 & \cdots & \tilde{\Gamma}_{p-2} & \tilde{\Gamma}_{p-1} \\ 0 & I & & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix}$$

and  $\tilde{\Sigma}$  can be determined from

$$\tilde{\Sigma} = \mathcal{A}\tilde{\Sigma}\mathcal{A}' + \tilde{\Sigma}_{\mathcal{E}}.$$

$\tilde{\Sigma}_{\mathcal{E}}$  is the usual estimator of the covariance matrix of  $(u_t'\beta, u_t', 0, \dots, 0)'$ . In JMulTi, the confidence intervals may be plotted for consecutive sample sizes  $\tau = T_1, \dots, T$ .

The recursive eigenvalues can also be used as the basis for formal tests of parameter constancy. Let

$$\xi_i^{(\tau)} = \log \left( \frac{\lambda_i^{(\tau)}}{1 - \lambda_i^{(\tau)}} \right)$$

and

$$\mathcal{T}(\xi_i^{(\tau)}) = \frac{\tau}{T} |(\xi_i^{(\tau)} - \xi_i^{(T)})/\hat{\sigma}_{ii}|.$$

Hansen and Johansen (1999) derive the limiting distribution of

$$\sup_{1 \leq \tau \leq T} \mathcal{T}(\xi_i^{(\tau)}).$$

Critical values for the limiting null distribution are tabulated by Ploberger, Krämer and Kontrus (1989). Stability is rejected if  $\mathcal{T}(\xi_i^{(\tau)})$  exceeds the critical value.

Another possible test statistic is based on the sum of the  $r$  largest recursive eigenvalues:

$$\mathcal{T} \left( \sum_{i=1}^r \xi_i^{(\tau)} \right) = \frac{\tau}{T} \left| \left[ \sum_{i=1}^r (\xi_i^{(\tau)} - \xi_i^{(T)}) \right] / \hat{\sigma}_{1-r} \right|.$$

Here  $\hat{\sigma}_{1-r}$  is an estimator of the standard deviation of the difference  $\sum_{i=1}^r (\xi_i^{(\tau)} - \xi_i^{(T)})$  which is based on the relevant part of the matrix  $\Upsilon + \Upsilon'$ . Again, Hansen and Johansen (1999) show that

$$\sup_{1 \leq \tau \leq T} \mathcal{T} \left( \sum_{i=1}^r \xi_i^{(\tau)} \right)$$

can be used for checking model stability.

In JMulTi different versions of these tests are implemented. If the short-term dynamics are assumed to be stable and a test of parameter change in the long-run part only is desired, one may first concentrate out the short-term parameters based on the full sample and then one may focus on recursive estimation of  $\alpha$  and  $\beta$ .

### 9.2.2 Implementation in JMulTi

The recursive eigenvalues panel is reached by selecting **Model Checking** → **Stability Analysis** → **Recursive Eigenvalues**. It is only available if a VECM without parameter restrictions and without exogenous variables is specified. Note, however, that exogenous variables are ignored if they are included in the original model and the Johansen estimation method is used. In that case, recursive eigenvalues of the model without the exogenous variables are reported. Several choices have to be made before displaying the graphs of the recursive eigenvalues and Tau ( $\mathcal{T}$ ) statistics.

**Start date** marks the beginning of the recursive estimation

**Sign. level for Tau-stability tests** a choice can be made from several different options; it determines the critical value shown in the graph

**Likelihood function estimation** if using **concentrated likelihood func.** is chosen, the short-term parameters are concentrated out on the basis of the full sample and only the long-run part is estimated recursively; if **recursively estimate all parameters** is selected, the short-run part of the model is also estimated recursively

**Eigenvalue CI estimation** shows confidence intervals of recursive eigenvalues; if the option **full sample estimates of standard errors** is chosen, the standard error  $\hat{\sigma}_{ii}$  is estimated on the basis of the full sample and not recursively; if the alternative choice **recursive sample estimates of standard errors** is made,  $\hat{\sigma}_{ii}$  is also estimated recursively based on the same subsamples as the recursive eigenvalues

### 9.3 Chow tests

In JMulTi, sample-split (SS), break-point (BP) and Chow forecast (CF) tests are available for VEC models. The SS and BP Chow tests for checking for a structural break in period  $T_B$  proceed as follows. The model under consideration is estimated from the full sample of  $T$  observations and from the first  $T_1$  and the last  $T_2$  observations, where  $T_1 < T_B$  and  $T_2 \leq T - T_B$ . Denoting the resulting residuals by  $\hat{u}_t$ ,  $\hat{u}_t^{(1)}$  and  $\hat{u}_t^{(2)}$ , respectively, and using the notation

$$\begin{aligned}\tilde{\Sigma}_u &= T^{-1} \sum_{t=1}^T \hat{u}_t \hat{u}_t', \\ \tilde{\Sigma}_{1,2} &= T_1^{-1} \sum_{t=1}^{T_1} \hat{u}_t \hat{u}_t' + T_2^{-1} \sum_{t=T-T_2+1}^T \hat{u}_t \hat{u}_t', \\ \tilde{\Sigma}_{(1)} &= T_1^{-1} \sum_{t=1}^{T_1} \hat{u}_t^{(1)} \hat{u}_t^{(1)'}\end{aligned}$$

and

$$\tilde{\Sigma}_{(2)} = T_2^{-1} \sum_{t=T-T_2+1}^T \hat{u}_t^{(2)} \hat{u}_t^{(2)'},$$

the BP test statistic is

$$\lambda_{BP} = (T_1 + T_2) \log \det \tilde{\Sigma}_{1,2} - T_1 \log \det \tilde{\Sigma}_{(1)} - T_2 \log \det \tilde{\Sigma}_{(2)} \approx \chi^2(k) \quad (\text{under stability}).$$

Here  $k$  is the difference between the sum of the number of parameters (including those in the residual covariance matrices) estimated in the first and last subperiods and the number of parameters in the full sample model, not counting the cointegration parameters in the matrix  $\beta$ . The null hypothesis of constant parameters is rejected if  $\lambda_{BP}$  is large.

Whereas the BP test checks if any of the parameters vary (except the cointegration parameters), the SS test is derived under the assumption that the residual covariance matrix  $\Sigma_u$  is constant and checks against the alternative that the other coefficients may vary. It has the form

$$\lambda_{SS} = (T_1 + T_2) [\log \det \tilde{\Sigma}_{1,2} - \log \det \{(T_1 + T_2)^{-1} (T_1 \tilde{\Sigma}_{(1)} + T_2 \tilde{\Sigma}_{(2)})\}]$$

and it is also used in conjunction with a limiting  $\chi^2$  distribution. The degrees of freedom are again obtained as the number of restrictions imposed by assuming constant coefficients

rather than a break in period  $T_B$ .

The CF statistic is

$$\lambda_{CF} = \frac{1 - (1 - R_r^2)^{1/s}}{(1 - R_r^2)^{1/s}} \cdot \frac{Ns - q}{KT_2} \approx F(KT_2, [Ns - q]),$$

where  $[x]$  denotes the integer part of the real number  $x$ ,

$$s = \left( \frac{K^2 T_2^2 - 4}{K^2 + T_2^2 - 5} \right)^{1/2}, \quad q = \frac{KT_2}{2} + 1, \quad N = T - k_1 - T_2 - (K - T_2 + 1)/2.$$

Here  $k_1$  is the number of regressors in each equation of the time invariant model and

$$R_r^2 = 1 - \left( \frac{T_1}{T} \right)^K |\tilde{\Sigma}_{(1)}| (|\tilde{\Sigma}_u|)^{-1}.$$

The CF test tests against the alternative that all coefficients including the residual covariance matrix may vary. It also rejects the null hypothesis of constant parameters for large values of the test statistic. The test is only available for full models without subset restrictions.

Because the actual small sample distributions of the test statistics under  $H_0$  (for models with time invariant parameters) may be quite different from the asymptotic  $\chi^2$ - or  $F$ -distributions (see Candelon and Lütkepohl (2000)), JMulTi offers bootstrap  $p$ -values. They are computed as described in the corresponding helpsection for VAR models. Now, of course, with cointegration relations estimated for the full sample period.

**Dealing with dummies** In practice, one often has regression models with a constant, where in addition several of the remaining regressor variables are dummies which might also be constant either for the first or for the second part of the sample created by splitting the sample at  $T_B$ . In order to avoid perfect collinearity between the regressors, JMulTi deletes dummies automatically from the estimation of either the first or the second part of the sample where appropriate with corresponding adjustment of the degrees of freedom if necessary.

### Specification in JMulTi

Chow tests can be performed for individual time periods or for a range of time points. In the latter case the results are given in tables as well as graphs.

### Input

**Break date** allows you to specify a break period for which Chow tests are to be performed

**Search over datapoints** selecting this option allows you to perform Chow tests not only for a single break date but over a range of the time points as specified in the adjacent menu entry

**Test range** allows you to choose the range for the **Search over datapoints** procedure

**Graph of bootstrapped p-values** checking this option produces graphs of  $p$ -values of the tests calculated for the break dates in the specified range

**Number of bootstrap replications** specifies the number of replications used to construct the empirical distribution function of the statistics. It has a substantial impact on the required computation time. For reliable results it may be necessary to select a few thousand replications although this may result in substantial waiting times

## Output

**break point Chow test (chow-bp)** value of BP test statistic

**boot p-val** shows the bootstrapped  $p$ -value of the corresponding statistic

**chi<sup>2</sup> p-val** shows the  $p$ -value of the approximating  $\chi^2$  distribution corresponding to the BP statistic

**df** degrees of freedom of the corresponding  $\chi^2$  distribution

**sample split Chow test (chow-ss)** value of SS test statistic

**boot p-val** shows the bootstrapped  $p$ -value of the corresponding statistic

**chi<sup>2</sup> p-val** shows the  $p$ -value of the approximating  $\chi^2$  distribution corresponding to the SS statistic

**df** degrees of freedom of the corresponding  $\chi^2$  distribution

**Chow forecast test (chow-fc)** values of CF test statistics, only available for models without subset restrictions

**boot p-val** shows the bootstrapped  $p$ -value of the corresponding statistic

**F p-val** shows the  $p$ -value of the approximating  $F$  distribution corresponding to the CF statistic

**df1** numerator degrees of freedom of the corresponding  $F$  distribution

**df2** denominator degrees of freedom of the corresponding  $F$  distribution, integer part of  $Ns - q$

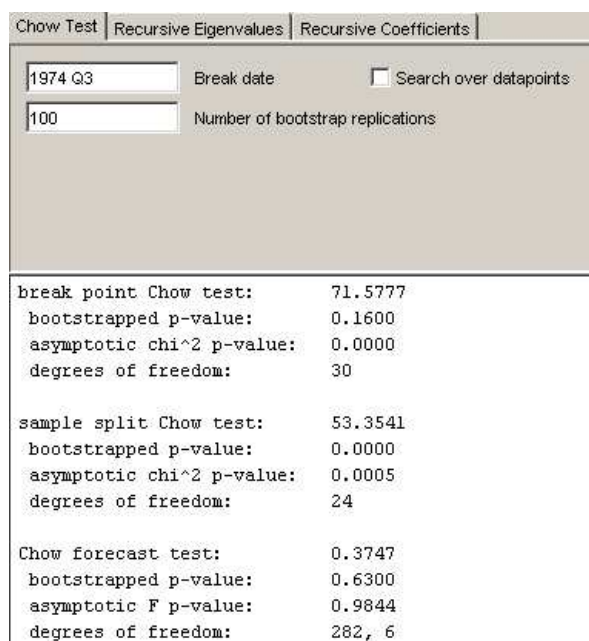


Figure 13: Chow tests for a single break point in a VECM.

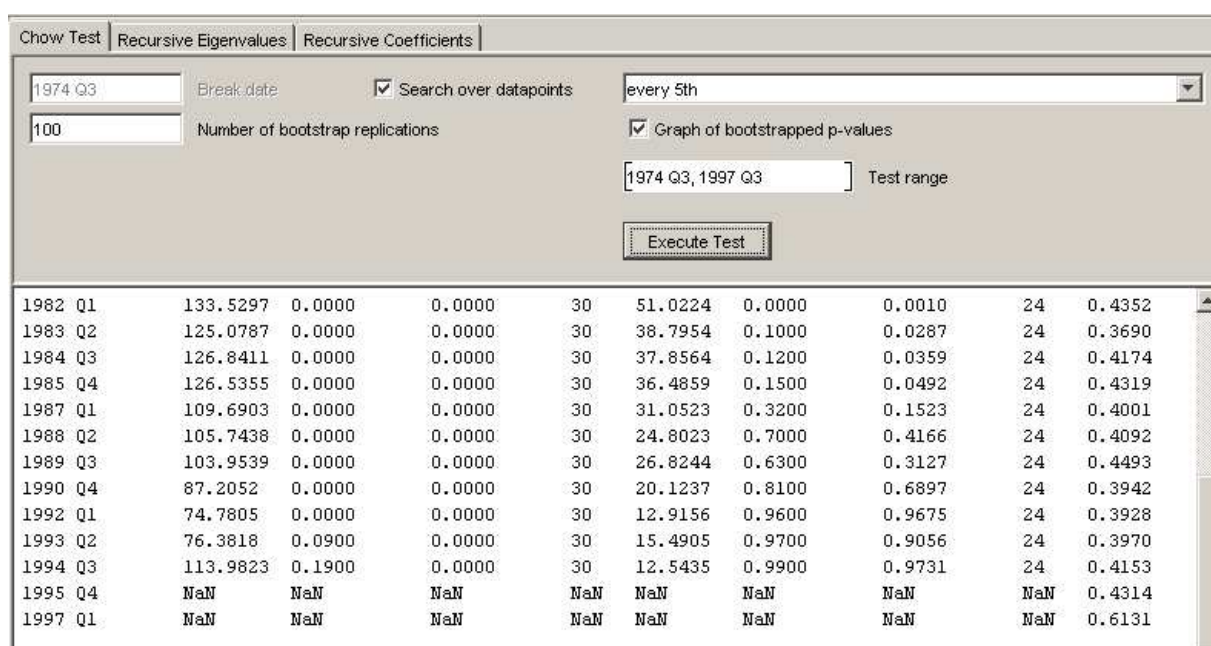


Figure 14: Chow tests for a range of possible break points in a VEC model.



## 10 Causality Analysis

### 10.1 Background

For VECMs two types of causality tests are implemented in JMulTi , tests for Granger-causality and tests for instantaneous causality. For both types of tests the vector of endogenous variables is divided in two subvectors,  $y_{1t}$  and  $y_{2t}$ , with dimensions  $K_1$  and  $K_2$ , respectively, so that  $K = K_1 + K_2$ . The subvector  $y_{1t}$  is said to be Granger-causal for  $y_{2t}$  if it contains useful information for predicting the latter set of variables. For testing this property, the levels VAR form (2) without exogenous variables of the model is considered. If that model contains  $p + 1$  lags of the endogenous variables as in (2), the test is based on a model with  $p + 2$  lags of the endogenous variables,

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \sum_{i=1}^{p+2} \begin{bmatrix} \alpha_{11,i} & \alpha_{12,i} \\ \alpha_{21,i} & \alpha_{22,i} \end{bmatrix} \begin{bmatrix} y_{1,t-i} \\ y_{2,t-i} \end{bmatrix} + CD_t + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

as proposed, e.g., by Dolado and Lütkepohl (1996). The null hypothesis that  $y_{1t}$  is not Granger-causal for  $y_{2t}$  is tested by checking the null hypothesis

$$\alpha_{21,i} = 0, \quad i = 1, 2, \dots, p + 1.$$

A Wald test statistic, divided by the number of restrictions  $pK_1K_2$ , is used in conjunction with an  $F(pK_1K_2, KT - n^*)$  distribution for testing the restrictions. Here  $n^*$  is the total number of parameters in the system (see Lütkepohl (1991)), including the parameters of the deterministic term. Of course, the role of  $y_{1t}$  and  $y_{2t}$  can be reversed to test Granger-causality from  $y_{2t}$  to  $y_{1t}$ .

The test is problematic if some of the variables are  $I(d)$  with  $d \geq 2$ . In that case the assumed asymptotic distribution of the test statistic may not be valid under the null hypothesis.

Instantaneous causality is characterized by nonzero correlation of  $u_{1t}$  and  $u_{2t}$ . Thus the null hypothesis

$$H_0 : \mathbf{E}(u_{1t}u_{2t}') = 0$$

is tested against the alternative of a nonzero covariance between the two error vectors in testing for instantaneous causality. The Wald test described in Lütkepohl (1991, Sec. 3.6.3) is reported in JMulTi .

### 10.2 Causality Analysis in JMulTi

Once a model without exogenous variables is specified, the causality analysis can be accessed by clicking on the menu **Structural Analysis**→**Causality Tests**. The endogenous variables are shown in the list. You can select from 1 up to  $K - 1$  variables. The respective  $H_0$  hypothesis

then appears on the panel and the tests can be performed by pressing the Execute Tests button.

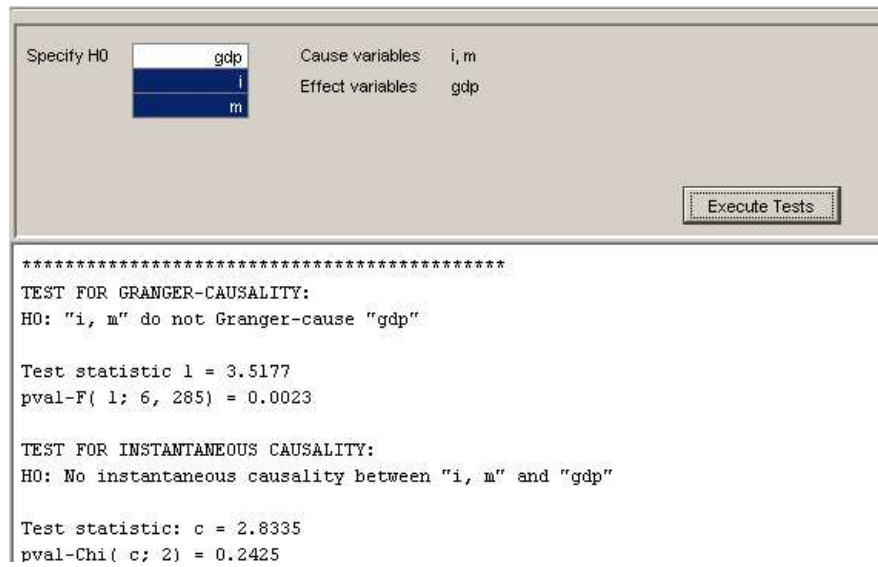


Figure 15: Causality Analysis for VECMs in JMulTi

## 11 Impulse Response Analysis

Impulse response analysis of VEC models is based on the levels VAR form and is implemented in JMulTi as for VAR models (see the Impulse Response Analysis for VAR Modeling).

## 12 Forecast Error Variance Decomposition

Forecast error variance decomposition for VEC models is based on the levels VAR form and is implemented in JMulTi as for VAR models (see the Forecast Error Variance Decomposition under VAR Modeling).

## 13 SVEC Analysis

The SVEC (structural vector error correction) model can be used to identify the shocks to be traced in an impulse response analysis by imposing restrictions on the matrix of long-run effects of shocks and the matrix  $\mathbf{B}$  of contemporaneous effects of the shocks.

The matrix  $\mathbf{B}$  is defined such that  $u_t = \mathbf{B}\varepsilon_t$  in (1) and, assuming that (1) is in reduced form, the matrix of long-run effects of the  $u_t$  residuals is

$$\Xi = \beta_{\perp} \left( \alpha'_{\perp} (I_K - \sum_{i=1}^{p-1} \Gamma_i) \beta_{\perp} \right)^{-1} \alpha'_{\perp}.$$

Hence, the long-run effects of  $\varepsilon$  shocks are given by

$$\Xi \mathbf{B}.$$

$\text{rk}(\Xi) = K - r$  and, hence,  $\Xi \mathbf{B}$  has rank  $K - r$ . Thus, the matrix  $\Xi \mathbf{B}$  can have at most  $r$  columns of zeros. Hence, there can be at most  $r$  shocks with transitory effects (zero long-run impact) and at least  $k^* = K - r$  shocks have permanent effects. Due to the reduced rank of the matrix, each column of zeros stands for only  $k^*$  independent restrictions.  $k^*(k^* - 1)/2$  additional restrictions are needed to exactly identify the permanent shocks and  $r(r - 1)/2$  additional contemporaneous restrictions identify the transitory shocks. For examples see Breitung, Brüggemann and Lütkepohl (2004). JMulTi has a facility to impose restrictions on  $\mathbf{B}$  and  $\Xi \mathbf{B}$ .

Estimation is done by maximum likelihood using the reduced form estimates obtained by the estimation method selected in the **Specify VEC Model** menu (see Breitung et al. (2004)). If the algorithm does not converge or problems with the estimation algorithm occur some manual adjustments may be necessary. Problems can be due to inappropriate restrictions.

Once an SVEC model has been estimated, **SVEC IRA** (impulse response analysis) and **SVEC FEVD** (forecast error variance decomposition) are activated and can be used.

### 13.1 Implementation in JMulTi

To access the SVEC analysis, estimate a reduced form VECM first and then go to **SVEC→Estimate SVEC Model**. Then impose the restrictions by clicking on the relevant elements of the matrices activated in the panel. The model will be estimated upon pressing the **Point Estimates Only** or the **Estimate with Boot. Std. Err.** button. In the latter case, bootstrap standard errors and corresponding  $t$ -ratios for the estimated elements of  $\mathbf{B}$  and  $\Xi \mathbf{B}$  are provided. Choose the desired number of bootstrap replications before estimation. If there are convergence problems, press the **Optimization Settings** button and adjust the settings in the panel.

Remember, however, that convergence problems can also be due to inappropriate restrictions.

As mentioned earlier, once the SVEC model has been estimated successfully, SVEC IRA and SVAR FEVD are activated. They can be accessed by going to SVEC→SVEC IRA or SVEC→SVEC FEVD.

## 14 Forecasting with VEC Models

Forecasting with VEC models is based on the levels VAR form and is implemented in JMulTi as for VAR models (see the Forecasting section under VAR Modeling).

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